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FINAL REPORT

SHORT PERIOD FLUCTUATIONS IN INTELLIGENCE

A Research Project Supported by a NASA Grant
in the Space-Related Sciences

NsG-518

Project No. DRI-614

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UNIVERSITY OF DENVER

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I. INTRODUCTION

The problem of isolating fluctuations in intellectual function is one that has long intrigued both psychologists and non-psychologists. We are all aware of our own short-period physiological, mood and motivational changes and many believe that their intelligence must undergo similar change. But existing scientific evidence (see Tyler, 1965 for a recent review) does not provide support for this kind of argument. It indicates that measures of intelligence--at least those intended for use with people other than infants--are usually highly stable over both short and rather extended periods of time: the changes which are indicated have almost always been regarded as errors of measurement. Thus in scientific developments (as opposed, perhaps, to common-sense discussions) intelligence usually has been regarded as a highly stable attribute of man.

A notable exception to the scientific theory arguing (albeit implicitly) that human intellect does not fluctuate over short period of time is that of Moran (1961) and his co-workers (Moran, Kimble & Mefferd, 1960; 1964; Moran & Mefferd, 1959). The work of these investigators has been directed at designing alternate-form tests to measure some of what are now regarded as the major, replicated-by-research dimensions of human intelligence, viz., what are known as the primary mental abilities (French, 1951; French, Ekstrom & Price, 1963; Guilford, 1966; Guilford & Merrifield, 1960). The Repetitive Psychological Measures (RPM) resulting from this work are intended to assess "change in performance with time" (Moran & Mefferd, 1959; p. 269). But while this is the purpose for which the tests are intended, little has been done to determine whether or not the performances actually assessed do, in fact, change in a reliable manner. Indeed, evidence somewhat contrary to an hypothesis of short-period change is provided by the high (relative to test length) test-retest correlations obtained over a period of 3 hours with the RPM tests (Moran & Mefferd, 1959; Table 2). Hence, while a basic hypothesis underlying this work certainly does imply that abilities change over short periods, evidence for this hypothesis either has not been developed or, where it has (as in test-retest correlations), does not provide convincing support for the hypothesis.

Questions about short-period change in human abilities are important, both scientifically and practically. Without presenting a complete and fully documented case to support this assertion, we can readily see some of the reasons why it is plausible. For example, since many theories about the nature of intelligence attempt to relate behavioral observations to underlying physiological processes and these latter are known to change over short periods of time, it is scientifically interesting to know which, if any, behavioral indicants of intelligence similarly change. From a practical point of view, it can be argued that if a particular kind of job, such as flying a spacecraft, demands that the people performing that job maintain a particular level of ability, then it is of considerable importance to know the range of fluctuation this ability is likely to have. In the example given (flying a spacecraft) information about fluctuations in abilities could be of use in selecting individuals to perform the job in question, in designing the equipment these individuals would use, in scheduling work loads for these people, etc. And this is but an example. There are, potentially, many applications for scientific findings showing short-period changes in human abilities.

Questions about short-period changes in human intelligence are important; yet very little research has been directed at answering these questions. Horn (1963b; 1966a; Horn & Little, 1966) has presented the view that one of the major reasons for this lack of research is that, until quite recently, there were no methods of analysis which would enable the researcher to distinguish between stable, virtually non-changing patterns of behavior and reliably observed patterns which, however, were fluctuating. The issues implied by this statement are very complex indeed: they relate to the whole of psychological theory in a quite fundamental way. Fortunately, however, as concerns the purposes of this study, we can examine most of the issues at a fairly general level and thus keep the complexity within manageable bounds.

First, let us recognize that several distinct meanings may be attached to a phrase like "short-period changes." As noted already, one meaning is simply unreliability--the fact that if measurement is sufficiently precise,[1] and even when we are careful to impose standard conditions in repeat measurements, there will be <u>random</u> fluctuations. Unreliability represents fluctuation which is not attributable to change in quantity of the attribute measured but attributable to measuring instrument, to the technician using the instrument and to similar influences. In this study we are concerned with unreliability, but only as it interferes with analyses of other kinds of change. Our aim will be to identify change which, in accordance with concerns for parsimony, is not likely to represent error of measurement.

A second meaning which may be associated with the expression "short-period changes" is the notion of a change which affects all (or nearly all) individuals in much the same way. For example, it is known (see Anastasi, 1958; p. 190-191 for a review of this evidence) that if test-naive subjects are repeatedly given intelligence tests, their scores tend to improve, at first markedly but progressively less as repeated testing continues. This effect has been ascribed to practice, to decrease in hampering emotional involvement with tests, to learning various "tricks" in marking answers and to several other factors. In such causal explanations the influences considered responsible for the observed changes are regarded as operating with respect to all (or most) people under observation or exposed to the experimental treatment. Moreover, these influences are regarded as affecting most people in much the same way--i.e., practice is not expected to cause one person to improve and another to get worse. In statistical analyses intended to establish effects of this kind, differences between subjects <u>within sessions</u> (groups) are treated as "error"--i.e., the variability against which average (over subjects) differences <u>between sessions</u> (groups) are assessed. But it is evident in this example that lawful change can exist and go undetected when change is identified in the manner indicated. For example, if it happened that one-half of a group of individuals became less emotionally involved and, for this reason, improved in performance as a function of experience with tests, but one-half of the individuals became more emotionally involved and (therefore) got worse in their performance, the net effect recorded by the above-mentioned procedures might well be "no change" and the lawful changes defined in the example would not be discovered.[2]

In the present study changes which affect all individuals in much the same way, as described above, will not be the principal concern. Instead the focus will be on individual differences in change. Similarly, the major purpose will not be to provide causal explanations for whatever change may be identified. Rather, the aim will be to establish the fact of concomitant variation in change, i.e., variation not ascribable to stable between-person differences. An example may help to make this point clear.

It has been established that various kinds of ability performances vary concomitantly between individuals tested on a single occasion. Horn & Cattell (1966a) have found, for example, that the performances representing the primary mental abilities labeled Induction (I), Figural Relations (CFR), Semantic Relations (CMR) and Associative Memory (Ma) do, in fact, vary together: persons who demonstrate much of one of these abilities tend to demonstrate much of the other abilities. Because this covariation exists, a factor identifiable by standard R-technique methods (Cattell, 1952) can be found repeatedly in studies that are properly designed and executed. This factor has been interpreted by Horn & Cattell (1966a) as representing what they call "fluid intelligence." Theoretically, it is conceivable that level of fluid intelligence--i. e., ability to perform in the various ways indicated above--fluctuates within individuals over periods as short as, say, a few hours. And it may be that the influences producing fluctuation do not operate in the same way, at the same times, with respect to all individuals under observation over a given period: as level of fluid intelligence goes up for some, it may go down for others. If this is true and it is true also, as seems likely, that the level of fluid intelligence for some individuals is consistently, despite fluctuations, above the level for other individuals, then to isolate the fact of reliable change such as is here described, it is necessary to have methods of analysis which enable us to identify: (1) patterns representing stable differences between individuals--patterns which will be referred to as <u>traits</u>--and (2) patterns representing between-person differences that are reliably observed on each of several occasions but which are not stable from one occasion to another--patterns which will be referred to as <u>states</u>. It is methods of this kind which have been developed only recently (Horn, 1963b; Horn & Little, 1966; Tucker, 1963; 1966). And it is with these methods, and the kind of change they are designed to reveal, that this study is principally concerned.

II. SUBSTANTIVE RATIONALE

The psychological concepts upon which this research is based were derived, for the most part, from a theory of fluid and crystallized intelligence (Cattell, 1941; 1957; 1963; Horn, 1965a; 1966b; Horn & Cattell, 1966a; 1966b; 1966c). This theory represents an attempt to integrate a considerable body of information and conjecture dealing with the development of human intelligence. In its latest form (Horn, 1965a) it runs to book length. Here, to get the present study properly focused, it is necessary to give particular consideration to those parts of the theory in which the fluid and crystallized concepts of intelligence are described.

The general theory states that in a representative sample of primary mental abilities there is concomitant variation representing two major kinds of attributes affecting performances in intellectual tasks. The two are somewhat independent in samples of older children and adults and thus can be separately identified in technically adequate factorial analyses of broad samples of primary mental abilities. Both attributes correspond closely to what is known semantically, and in general psychological theory, as intelligence. That is, both involve the processes of relation-perceiving and correlate-educing which Spearman first identified as integral to intelligence. And both involve other processes--such as concept attainment, abstracting, temporal integration--since shown to be representative of what is usually regarded as intelligent behavior. Hence, both may be referred to as kinds of intelligence. For reasons that are not crucial for present purposes, the terms "fluid" and "crystallized" are used to designate the two kinds of intelligence.[3]

Although Gf and Gc are similar in several important respects, they differ in terms of manifested patterns of ability performances, in terms of developmental factors producing these patterns and in terms of the influences which affect immediate display of the patterns. Fluid intelligence is manifested primarily in tasks wherein the materials can be seen to be culture fair relative to other materials used in the measurement of abilities. The fundaments of such tasks are either novel for most persons being measured or else are extremely common, overlearned elements of the culture of these people, and the generalized solution instruments (also termed "aids" See Cattell, 1963; Horn, 1965a) required for problem solutions are not of the kind made available by only favored educational opportunity. Thus fluid intelligence is identified in tasks requiring relation-perceiving, reasoning, abstracting, etc., in the immediate testing situation and in materials with which most people tested would be familiar. In contrast, crystallized intelligence, although it, too, involves reasoning, abstracting, etc., in the immediate situation, involves tasks which require the person to use the relatively abstruse concepts and aids derived from the collective experience which defines a culture.

The measurement distinction between Gf and Gc can be made clearer with an example.

Thus consider the following analogic reasoning problems:

Broom-Floor::Spoon-	Fork	Table	Soup	Dish
Hippocrates-Galen::Aeschylus-	Greece	Euripides	Pericles	Plato

Both are to some extent ambiguous, in the sense that one might adequately defend more than one answer, and yet it is evident that both require ability to perceive relations. But the first problem involves fundaments--i.e., concepts represented by words--with which most adults in this country would be familiar, whereas the second item contains fundaments with which many adults could not be at all familiar. Yet it is clear, too, that if one knows the referents for the words in question, the analogies are of about equal difficulty. Thus both problems allow for measurement of analogic reasoning ability, but in using the second problem we measure also (to a greater extent than in the first problem) a component of intelligence representing degree of acculturation. When this latter component is found in analogic reasoning, induction, etc., the result is crystallized intelligence. It is in this sense, then, that Gc is a dimension indicating the extent to which one has appropriated for his own use, as it were, the collective intelligence of a culture, whereas Gf, involving many of the same basic processes, does not so fully represent this kind of appropriation.

The development of both Gf and Gc depends upon the conditions of various underlying physiological structures, including, principally, neural tissues but not excluding sensory organs, motor pathways and other such units involved in the organism's processing of information for the intellect. Thus, both Gf and Gc reflect, in part, a history of influences deriving from heredity and unfolding in maturation. Similarly, both reflect a history if injuries, illnesses and similar influences directly affecting physiological structure and process. It is therefore not correct to say that Gf is the only representative of hereditary-physiological influences in the development of intelligence. Both Gc and

Gf are outcomes of the operation of such influences. But because fluid intelligence is most closely tied to expression of ability in the immediate situation and is less fully supported by the elaborate cell assembles and phase sequences (Hebb, 1949) associated with build up of cultural concepts and aids, Gf, in contrast to Gc, is the purer behavioral representation of on-going neural-physiological function. On this basis it can be predicted that injuries to the physiological structures which support display of intelligence will have a greater immediate influence on Gf than on Gc. On this basis, too, it can be predicted that insofar as fluctuating physiological changes are manifested in the behaviors which define intelligence, there should be greater short-period fluctuation in Gf than in Gc.

Support for several provisions of the Gf-Gc theory has come from recently completed studies. Cattell (1963), Horn (1965a), Horn & Bramble (1966) and Horn & Cattell (1966a) have found that patterns representing fluid and crystallized functions do, indeed, appear in factorial analyses of samples of primary mental abilities. Horn (1965a) and Horn & Cattell (1966b; 1966c) have found that the level of these functions differs for different age groupings in a way predicted by the theory. In performances representing the primary abilities found to define Gf, they found significant differences favoring young adults; in performances representing the primaries found to define Gc, the significant differences favored the older adults; in primaries allowing about equally for use of either Gf or Gc, there were no significant differences between age groupings.

These findings thus provide incentive to explore further implications of the theory. One set of implications has to do with the hypothesis stipulating that Gf should manifest greater short-period fluctuation than Gc. There is no evidence to provide a firm basis for either acceptance or rejection of this hypothesis. One of the major purposes of this research is to produce information bearing on this point.

The recent research mentioned above, particularly that of Horn (1965a) and Horn & Cattell (1966a), indicated general factors in addition to those described as fluid and crystallized intelligence. Chief among these other factors was one representing a general visualization function. This produced variance in all primaries in which the subject was allowed to, or required to, visualize relationships in order to solve a problem. Tasks defining the factor included some in which the subject would need to imagine movements of objects in space; some in which he would need to find a particular configuration imbedded within other configurations;

others in which he would need to bring about closure among disparate parts of a configuration; and still others in which he would need to quickly scan several configurations to locate one designated in instructions. Thus it was evident that a central process involved in the various performances is one having to do with visualizing.

But now the important point for present purposes is that the tasks which help to define this general visualization function (abbreviated Gv) also help to define the fluid intelligence function. Indeed, Gf and Gv were highly cooperative (Cattell, 1952) in the research mentioned above; careful rotation was required to effect a clean separation of the two. The fact that the two are cooperative means that unless proper precautions are taken, evidence supposedly relating to Gf can, in fact, pertain to Gv and vice versa. In other words, unless the variance associated with Gv is accounted for, results supposedly showing function fluctuation measurable through the tasks defining Gf can indicate change in the visualization process also measured in the tasks in question. And this kind of reasoning applies with respect to the other general functions mentioned above. An investigator must keep aware of the fact that his measures are complex indicators of the basic processes to which his research refers and that control over major factors other than the one of principal concern is essential if relatively clear findings are to emerge.

Besides general visualization and the Gf and Gc functions, the principal factors operating at the second order among primary abilities appear to be a general speediness (abbreviated Gs), a broad fluency dimension (abbreviated F), and a general carefulness function (abbreviated C).

General speediness is identified primarily in simple clerical tasks which do not involve visualization or intelligence to any considerable degree, although some variance on this factor is indicated for virtually any speeded test.

It might seem that general speediness would be equivalent to general fluency, for this latter definitely indicates speed of performance. However, F is shown most clearly in tasks where the speediness appears in the production of words, word parts and ideas which must be phrased in words (in contrast to, say, images, i.e., "visual ideas"). This factor is cooperative with Gc and in this sense seems to represent a kind of "store of knowledge" function, the implication being that if the "store" is large, more elements can be quickly taken from it. However, existing evidence does not rule out the possibility that F represents, not size of a "store of knowledge", but merely speed of transmission of elements in this "store" to the production modalities involved in writing and speaking.

It could be argued, too, that general speediness and general carefulness are merely opposites on a single dimension. However, the Horn-Cattell results definitely suggest that the two are relatively independent when seen in a broad sample of ability performances, although the correlation between the two is non-zero and negative, as expected. General carefulness represents an unwillingness to give a wrong answer to an item demanding intellectual ability, and the evidence here referred to suggests that this "unwillingness" can vary largely independently of the speediness with which one can do simple clerical tasks. A tentative interpretation of C is that it represents development of the superego.

It must be noted that the nature of the Gv, Gs, F and C functions was not at all clearly understood at the time of undertaking this research: a theory which would relate these processes to short-period fluctuations had not been very well worked out. The theory developed since that time is not much further advanced.

McFarlane Smith (1965) has provided a rather extensive analysis of spatial abilities and visualization functions. His work indicated that spatial abilities are often prominent in men of genius in the physical sciences and mathematics. However, Smith was not always careful to distinguish between the influences of Gf and those of Gv, so that one is left wondering whether it is visualization, per se, which characterizes outstanding scientists and mathematicians, or whether it is mainly fluid intelligence expressed in visual symbols.

Throughout Smith's treatment there was implicit acceptance of an hypothesis implying that spatial abilities are almost exclusively trait-like and thus show little or no function fluctuation. Yet, it is intuitively reasonable to suppose that a person's ability to visualize does, in fact, fluctuate considerably, depending upon conditions of rest, fatigue, diet, and so on. On this intuitive basis, although Gv was included in the study primarily for the purpose of distinguishing its effect from that of Gf, the hunch (not to dignify it with the term hypothesis) was that Gv would show considerable fluctuation over short periods of time.

If F represents mainly a "store of knowledge" and does not relate very closely to speed of transmission from this "store", then it would not be expected to change very much over short periods of time. If the opposite were true, however, then much state variation would be predicted, whereas if F represented a combination of these two kinds of processes, as seems most likely, there would be evidence of considerable stability of the function coupled with some function fluctuation. This study can perhaps provide us with a basis for a more definitive choice from among these alternatives.

On first consideration, it might seem that general speediness would almost certainly represent a state-like function, such as striving in the immediate testing situation. But on closer consideration it becomes evident that, theoretically, even such striving could be mainly trait-like. Similarly, general carefulness can be viewed either as mainly a state-like function (e.g., a variable test-taking strategy) or as mainly a trait-like function (superego structure). However, these factors are considered in this study primarily for the purpose of developing hypotheses for future research.

III. METHODOLOGICAL RATIONALE

As was stated in the introduction of this report, a particular conception of change is to be given primary consideration in this study. This conception is one implying concomitant variation of several variables, but variation which is not accounted for by the fact that one individual is consistently different from another. Looked at in terms of our commonly-used R-technique breakdown of variance into components, this conception may be represented by partitioning some R-technique components into two further components, viz., trait and state components, and by recognizing that some R-technique components could, theoretically, represent state variation alone. Using the raw data representation shown in Figure 1, these ideas may be summarized as follows:

$$\begin{array}{lcccccccc} x_{jki} & = & \underbrace{(t_{j1}T_{1i} + s_{jk1}S_{1ik})} & +\ldots+ & \underbrace{(t_{jk1}T_{1ik})} & +\ldots+ & \underbrace{s_{jkm}S_{mki}} & +\ldots & \\ x_{ji} & = & a_{j1}F_{1i} & +\ldots+ & a_{j1}F_{1i} & +\ldots+ & a_{jm}F_{mi} & +\ldots & \end{array} \qquad (1)$$

where x_{jki} represents an observed score of person $\underline{i}$, as obtained with test $\underline{j}$ on occasion $\underline{k}$, and the symbols on the right represent the factors into which the observed score may be partitioned. In the lower section is a specification equation like that associated with traditional R-technique, single occasion factor analysis, in which it is not necessary to indicate occasion, $a_{jm}(m = 1, \ldots, M)$ is the factor coefficient, representing the average (over subjects) extent to which test $\underline{j}$ involves factor $\underline{m}$, and F_{mi} is the factor score, representing the quantity of factor m characterizing person $\underline{i}$. In the upper section, then, is a similar kind of specification equation, but one in which it is necessary to designate the occasion, and where t_{jq} represents the average (over both subjects and occasions) involvement of test $\underline{j}$ in trait factor q, T_{qi} is a factor score representing the quantity of the <u>trait</u> q possessed by person $\underline{i}$, s_{jkr} is a situational factor coefficient representing the extent to which test $\underline{j}$ measures state factor $\underline{r}$ on the particular occasion $\underline{k}$, and S_{jkr} is a situational factor score representing the level of the <u>state r</u> in person $\underline{i}$ on this particular occasion $\underline{k}$.

In both models it is assumed that what is referred to as common factors can be identified in actual experimental analyses by the fact that several variables involving a particular factor covary in the manner implied by the calculations of factor analysis. In traditional factor analysis this implies that on a given occasion persons with high scores,

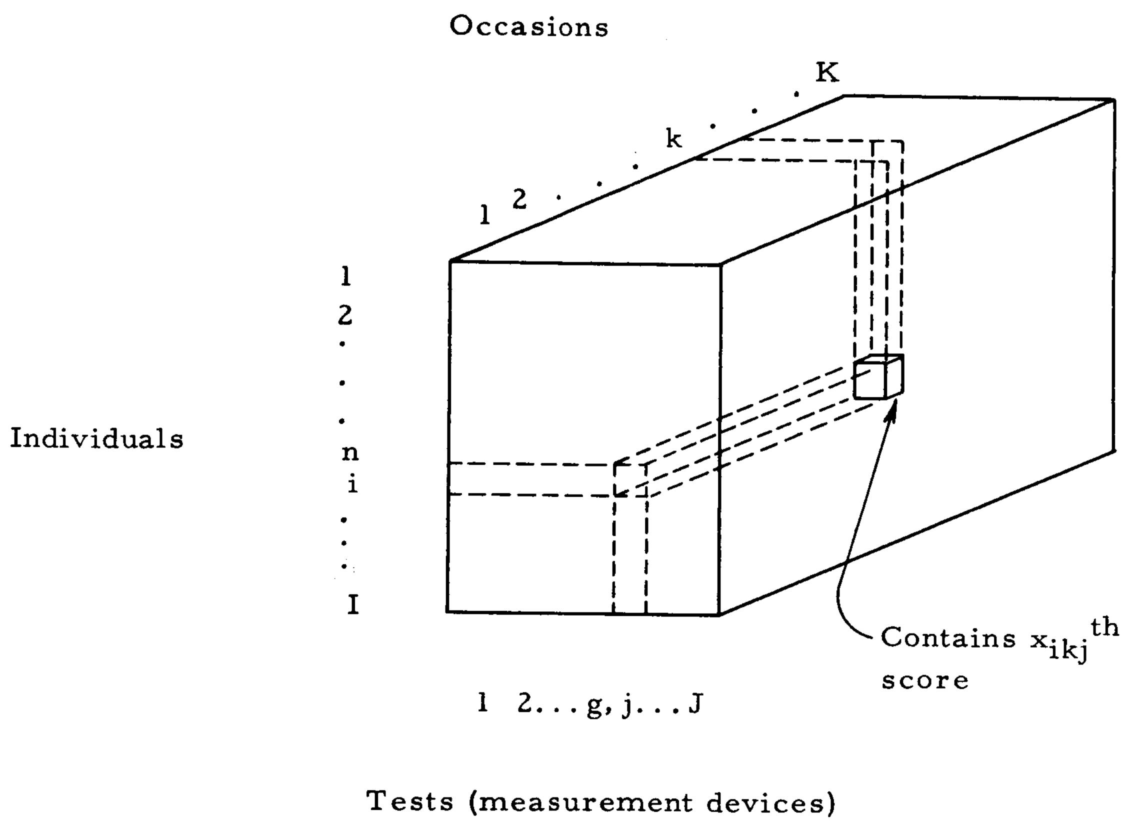

Figure 1. The Data Box: A Representation of Measurements Obtained in the Study of Change and Invariance

say, on one test tend to have high scores on the other tests in a factor defined, in part, by the first-mentioned test. In the model being developed here it implies this and more. It implies that in defining trait factors, persons with high scores on the tests of a particular factor on one occasion tend to have high scores on these same tests on other occasions. In defining state factors the implication is that reliable covariation is left after that ascribable to trait factors has been partialed.

A set of statistical-mathematical procedures for dealing with this kind of model has been developed by Horn (1963b; Horn & Little, 1966), using derivations from multiple-discriminant analysis (Anderson, 1958; Rao, 1952). The principal rationale for these procedures can be stated in fairly brief terms, namely: either occasions or tests or subjects can be regarded as the "groups" involved in discriminant analysis and factors can be defined in such a way that the linear combinations these imply will maximally (in a least squares sense) discriminate between groups; these factors may then be partialed from the original covariance matrices, after which the resulting residuals can be pooled and factored. In more detail the rationale is as follows:

Consider a test $\underline{j}$ given repeatedly over $\underline{K}$ occasions to a set of I subjects. Now a linear combination

$$x_{ij} = x_{ij1} + x_{ij2} + \ldots + x_{ijK} \tag{2}$$

of the scores obtained with this test by subject $\underline{i}$ on the various occasions will have non-zero internal consistency to the extent that the scores obtained on any one occasion covary with the scores obtained on other occasions. Thus, since the correlation of a variable with another variable is limited by the internal consistency reliability of that variable, linear-combination scores of this kind for test $\underline{j}$ can correlate with similar linear-combination scores for $\underline{g}$ to the extent that: (1) the scores for each test, considered separately, covary from one occasion to another, and (2) the scores for one test covary with the scores for the other test. The important point here is that the correlation between the two linear composites can tend to zero even when the correlations, r_{jgk}, on <u>all</u> of the $\underline{K}$ separate occasions are high and have the same sign. This means that correlations between linear-composite measures like those of equation (2) give crucial information about traits.

Let us, therefore, consider forming a J by J matrix of intercorrelations among J linear-composite variables of the above kind. A typical element in this matrix may be symbolized:

$$t_{jg\cdot} = \frac{1}{I}\sum_{i}^{I} z_{ij\cdot}\ z_{ig\cdot} \tag{3}$$

where the use of *z* indicates that the linear-composite variables have been standardized, thus bringing the mean of the means for all variables to zero and this overall standard deviation to 1.0. This violates no basic assumption implicit in the use of most psychological variables and does not eliminate variable levels of importance for the model being developed here. The use of *t* to symbolize the correlations in this case is intended to provide a mnemonic aid, since these coefficients may be thought of as representing the variation essential for the identification of traits. The entire matrix of these coefficients is conveniently designated T. If traits, thought of as factors among variables, do not exist, this matrix will tend to identity form, whereas to the extent that consistent trait influences do operate, this matrix will contain non-zero elements in the off-diagonal places and the factors for this will be the same as the factors among correlations obtained on any given occasion.

Next consider each subject separately and the sample of *K* occasions upon which each is observed. Using this sample of occasions, we may compute what is usually referred to as the p-technique correlation for subject *i*, thus

$$w_{ijg} = \frac{1}{k}\sum_{k}^{K} z_{ijk} z_{igk} \tag{4}$$

where, again, the use of *z* indicates standardization, this time over occasions for a particular subject. This standardization means that the over-occasion scores (trait levels) and variabilities from occasion to occasion are made the same for all subjects for which this kind of coefficient is computed. Thus the coefficient will be non-zero to the extent that score changes from occasion to occasion on the test *j* are accompanied by similar changes in score on the test *g*. Notice that a non-zero coefficient of this kind can occur either when: (1) the scores for subject *i* are consistently above the scores for subject *h*, as in the case where *j* and *g* measure stable trait and *i* has more of this than *h*, or (2) the scores for subject *i* are sometimes above and sometimes below (i.e., inconsistently or randomly above and below) the scores for subject $\underline{h}$, the case expected when some reliable state variability exists. In either case, however, there must be, if the w_{ijg} coefficients are to be statistically significant, reliable change on the variables in question and change in score on one variable must be consistently accompanied by similar change on the other variable. In other words, if changes in scores on tests $\underline{j}$ and $\underline{g}$ represent only errors of measurement, then the w_{ijg} coefficients will tend to zero: more correctly, if we consider the I such coefficients for all subjects, the expected mean for the distribution of these coefficients is zero and the standard deviation for this distribution should approximate the standard error for correlations based upon sample size $\underline{K}$ in the case where the correlation in the population is zero. Thus, if the observed variability indicates only error of measurement, then $w_{.jg}$ coefficients formed by pooling over subjects:

$$w_{.jg} = \frac{1}{IK}\sum_{i}^{I}\sum_{k}^{K} z_{ijk}z_{igk} \tag{5}$$

will be approximately zero and to the extent that this is true for all variables under consideration, a J by J matrix, W, of such coefficients will tend to identity form. In contrast, to the extent that a set of tests reliably measures the same state, whether or not this is also a trait, the $w_{.jg}$ coefficients for these tests will be non-zero and W will not have identity form. This can occur: (1) when the tests in question measure pure states--the condition existing when it is purely a random matter for one subject to be above another in scores on the various tests defining the factor, or (2) when the tests measure traits but traits which have some reliable function fluctuation--one subject tends to be above another, but reversals in this ordering do occur and they occur consistently with respect to several tests involved in the factor.

The T and W matrices defined above go a long way toward providing us with basic summary statistics upon which to base multivariate analyses aimed at disclosing the nature of change and invariance in a set of variables. Factors derived from the coefficients of T provide a firm basis upon which to base an inference about traits and the factors obtained from W indicate state patterns if such exist. But there is likely to be some confounding of state with trait and vice versa in these identifications. This is true mainly because in actual applications K is likely to be small, for various practical reasons, and occasions are not likely to be as independent as was implied above in developing the model. If tests $\underline{j}$ and $\underline{g}$ measure only a pure state (e.g., thirst), but adjacent occasions are not sufficiently separated in time to allow the state to dissipate in one individual and build up in another, then the T matrix will contain some covariation indicating this fact and likely to be misinterpreted as trait variance. But now this kind of consistent variability is also picked up in the W matrix. Hence, it can be argued that if this variability recorded in W is eliminated in T, the factors then resulting will provide a purer basis upon which to base an inference of trait. A statistical rationale based upon this kind of reasoning follows from a consideration of the principles of discriminant analysis.

A set of occasions for a particular subject may be viewed as a "group", analogous to a group in the usual developments and applications of discriminant analysis. That is, just as we can regard K school boys as a group distinct from a group of K school girls, so one individual observed on K occasions may be regarded as a "group" of himself, so to speak, distinct from a "group" of a second individual observed on K occasions. The J measures obtained on these one-person "groups" then constitute a basis for describing differences between "groups".

A discriminant is a linear combination of measures which best (in a least squares sense) separates (i.e., discriminates between) groups despite variation within the groups. A discriminant among "groups" comprised of repeat observations on each particular individual is thus a linear combination which best separates individuals despite variation within individuals. This is the essence of the definition of trait developed above. Hence, discriminants among one-person "groups" constitute a basis for definition of traits, as is argued in somewhat greater detail elsewhere (Horn, 1963b; Horn & Little, 1966).

The result of deriving discriminants, following the usual procedures (see Horn, 1963b; Horn & Little, 1966), is to determine principal components (i.e., factors) on a T matrix modified by being premultipled by the inverse of a "within-subjects" dispersion matrix of the form of the W matrix defined above; that is,

$$B_1 = W_1^{-1}T_1 \tag{6}$$

where it can be seen that if W happened to equal T, the resulting B_1 would be an identity matrix. Thus, to the extent that the pooled covariation within subjects differs from the between-subject covariation among average scores, the off-diagonal elements of B_1 will be non-zero and there will be common factors. By the above reasoning, these factors are mainly indicative of stable trait influences operating among the variables.

The subscripts on the symbols in equation (6) are meant to remind us that both the W and T matrices might be defined, not as correlation matrices, but as either covariance matrices or as sums of squares and sums of cross-products matrices. Thus there would also be defined

$$B_2 = W_2^{-1}T_2 \tag{7}$$

when covariance matrices were involved, and

$$B_3 = W_3^{-1}T_3 \tag{8}$$

when cross-products matrices were used.

In B_1, trait level differences and differences in variability from occasion to occasion would be eliminated by standardization, thus leaving covariation in change alone to determine the discriminant factors. In B_2, differences in variability between sessions would be allowed to influence the determination of the discriminant factors and in B_3 both these differences and differences in level would operate. Often it would be worthwhile to consider the factors resulting when these latter influences were allowed to operate. In the study reported here, however, the measurement devices used on separate occasions were not precisely parallel forms, so the differences in level and variability referred to above were, in part at least, arbitrary, reflecting the somewhat different difficulty levels and internal consistencies of different forms of the tests. Because of this, the analyses were carried out only with the matrices referred to in the definition of equation (6).

The correlation between two tests on any <u>one</u> of several separate occasions can be non-zero because either (1) the same stable trait is measured by both tests, or (2) the same pure state is measured by both tests, (3) the same trait and the same state (function fluctuation of a trait) are measured by the two tests, or (4) influences specific to the occasion operate. The contribution of stable trait to this correlation is represented in B_1 and the F_t factors defined on this. Hence, if this contribution to the correlation is subtracted out, the resulting residual should contain covariance representing the other three influences.

Pooling matrices for separate occasions to eliminate random variation across occasions and factoring the residual defined by subtracting B_1 should, therefore, provide evidence about states and influences specific to occasions.

IV. PROCEDURES

1. Operational Definitions of Variables

The prior research mentioned above established a basis for selection of variables but did not provide a complete sample of the measurement devices needed for this study. This is because most of the prior research was based upon single-occasion testing. Only rarely did it result in the construction of more than two forms of a test found to define a factor. Therefore, one of the first major tasks of this research was to construct several alternative forms of the tests which previous research had established as probable markers for the general factors of principal concern.

In Table 1 are listed the tests actually developed for this research. The titles for the tests are meant to be descriptive of the performances involved and are not necessarily the same as the titles used for the same tests in other studies. The column designated "source" in this table includes either the name of the person who invented the test or, if not this, the name of the person who developed most of the items incorporated into the tests finally developed.

In the "Primary Factor" column are listed the currently used titles for the primaries which tests represent. These titles and the symbols in the next column were taken from either French (1951), French, Ekstrom and Price (1963) or Guilford and Merrifield (1960).

The reliabilities listed in the "Avg. r_{xx}" column are Spearman-Brown corrected split-half coefficients averaged over 3 of the 10 sessions. Two separately timed sub-forms for each test were given on all ten sessions. For the first, fifth and tenth sessions the two separate forms were intercorrelated. For each particular set of two forms the resulting three intercorrelations were averaged. Then, because the scores for the two forms were added together to provide the score actually used in analysis, the Spearman-Brown formula was applied to the average correlations to give an estimate of the reliabilities of the full-length variable.

The "reliabilities" obtained in this way are, of course, internal consistencies, not stability coefficients. Also, since the separate subforms of the tests were put together somewhat arbitrarily and cannot, therefore, be expected to be perfectly parallel in the psychometric sense

Table 1

Description of Tests and Primary Factors

	Test Name	Source	Primary Factor	Symbol	Avg. r_{xx}	Scoring	Number Items	Work Time (Secs.)
1.	Encircling Numbers	Moran	Perceptual Speed	P	71	R	475	90
2.	Placing Dots	Moran	Aiming	A_i	90	R	300	90
3.	Following Paths	Moran	Visualization	Vz	93	R	80	180
4.	Copying Designs	Moran	Flexibility of Closure	Cf	81	R	36	130
5.	Figure Series	Cattell	Figural Relations	CFR	62	R	6	150
6.	Dominoes (D48)	Black	Deduction	D	70	R	6	150
7.	Remembering Digits	Horn	Memory Span	Ms	85	R	14	80
8.	Remembering Designs	Graham	Memory Span	Ms	52	K	2	120
9.	Letter Series	Horn	Induction	I	84	R	7	200
10.	Verbal Analogies	Horn	Semantic Relations	CMR	76	R	8	180
11.	Remote Associations	Mednick	Originality	O	48	R	10	150
12.	Controlled Associations	Thurstone	Associational Fluency	F_a	77	R	2	90
13.	Things	Taylor	Ideational Fluency	Fi	65	R	1	180
14.	Vocabulary	Horn	Verbal Comprehension	V	58	R	10	150
15.	General Information	Horn	Verbal Comprehension	V	67	R	6	150
16.	Irrelevant Associations	Horn	Carefulness Association	F_r	71	W	2	130

of this term (Gulliksen, 1950), the reliability estimates should be regarded as slight under-estimates of the true internal consistency reliabilities. It follows that in some cases the communalities for variables can be expected to be larger than the estimated reliabilities.

In the "scoring" column of Table 1 are listed several symbols to indicate the operations whereby a score was obtained for a subject. "R" means that the number of responses judged "correct" is the score. In a very simple test, such as "Encircling Numbers," this is the same as the number of responses made in accordance with directions. In other words, if the task was to draw a circle around all number 9's in a row of numbers, score was the number of 9's encircled. In this example no credit would be given if the number 8 were encircled, but no points would be subtracted for this failure to follow directions.

In a complex test, such as analogies, an answer judged correct represents the reasoning of the test constructor and is somewhat arbitrary for this reason. For example, in the scoring of the analogy:

Hippocrates-Galen::Aeschylus- Greece Euripides Pericles Plato

it is assumed that the essential relationship between Hippocrates and Galen is that of the occupational activity for which they are mainly remembered (viz., both were physicians of a sort) and that the choice of answer should depend upon perception of a similar relationship between Aeschylus and one of the choices. Aeschylus is mainly remembered for his activities as a playwrite and Euripides is mainly remembered for similar reasons; therefore Euripides is judged to be the "correct" answer and the other choices are judged incorrect. However, it is apparent that there are other relationships which might be considered in answering this kind of item. For example, both Hippocrates and Galen are listed as born in an even-numbered year (460 and 130 respectively). Aeschylus is said to have been born in an odd-numbered year (525) and, of the choices, only Plato is said to have been born in an odd-numbered year: hence, on this basis Plato might be selected as the answer. But this answer would be judged wrong on grounds that the reasoning here outlined would only rarely be used and in most cases when the answer "Plato" was given, it would represent a guess, rather than perception of a relationship. The person who arrives at this "wrong" answer by correct reasoning is penalized and this results in some loss of reliability and validity of measurement. But this kind of mistake probably does not occur very often or very consistently with respect to any particular subject, so the score obtained over several items can have creditable reliability and validity.

The letter "W" in the "scoring" column indicates that a score was obtained by counting the number of responses judged to be "wrong" or irrelevant. For example if in the Controlled Associations test one responded with the word "Fly-paper" to the key word "Square, " the answer would be judged "wrong." Three scorers worked on the tests and in situations like this one, the response was discussed to obtain a consensus as to whether or not it should be scored wrong.

The letter "K" in the scoring column indicates that the Memory-For-Designs test was scored according to a key based upon a different rationale than those outlined above. The instructions for scoring this test are provided in a manual prepared by Graham and Kendall (1960).

In the last two columns of Table 1 are listed the numbers of items and work times for the single-occasion forms of the tests. The time needed for instructions was variable, depending upon the session, and of course, "number of items" on the highly speeded tests does not mean the number that really could be attempted. In the Placing Dots test, for example, 300 small circles were presented, but no subject could really be expected to locate a dot in each of these circles in the time allowed (90 seconds).

Where more than one test was used to measure a primary factor, the scores for the separate tests were converted to standard score form and added together to provide the primary measure that was used in analyses. The resulting measures were then standardized separately within each session. This was done because although differences in the means and sigmas for different occasions might be significant (when analyzed by means of an analysis of variance, for example), the interpretation of these differences could not be unambiguous. The primary factor tests, although conceptually parallel, were not constructed in a way to ensure that they would necessarily be psychometrically paralled (Gulliksen, 1950). Hence significant differences between occasions could represent only the fact that forms of a test varied in difficulty or discriminability: such differences need not represent any change in the subjects measured. Also, since (as discussed in sections I and III) the principal purpose of this study was not to study changes common to all (or most) people of a group but to classify subjects according to patterns of change, the overall differences between sessions could be safely eliminated.

2. Data-gathering Procedures

The measurements indicated above were obtained on 10 separate occasions for each of 106 male inmates at the Colorado State Penitentiary. The tests were administered to groups of about 10-15 subjects. The first testing session began at about 9:30 A.M. on Monday of a given week; the second session began at about 1:00 P.M. on the same day. Eight sessions followed - one in the morning and one in the afternoon (at about the times indicated) on each day, Tuesday through Friday. The first testing session lasted approximately 2 hours. The sessions after this were somewhat shorter due to the fact that instructions could be given more quickly after the men had gained initial familiarization with the tests.

The men who completed all sessions were paid $2.00 for their efforts. Only two of those who started the testing failed to complete all sessions. Motivation appeared to be quite good throughout the testing, although it seemed to lapse slightly in mid-week relative to the other days and in afternoons relative to mornings. However, these were only impressions, not observations corroborated by analyses of data.

The generally high motivation was no doubt due in part to the financial incentive offered for completion of the tests. Two dollars may not seem like much for the amount of work done, but in the economy of the prison it is a good deal more than it seems to be when considered within the economy outside the prison. The jobs a prisoner can work at to provide funds for cigarettes, books and magazines, gifts, etc., pay approximately 10 cents per day. Many of the men get very little, or no, money from outside the prison. The men who participated in this study did not lose the income from their regular jobs, so the two dollars they earned by doing the tests was a kind of bonus.

But motivation was good, also, because the men (for the most part) were keen to contribute to a study that might help to provide a better understanding of human behavior. Most of the inmates in prisons are aware that they have problems which might be less troublesome were the sciences and technologies of psychology and related disciplines further advanced. Often, therefore, they want to aid such advance.

There are several other, somewhat related, reasons why co-operation in convict samples is generally high. Suffice it to say here that lack of motivation was not a problem in this study in spite of the fact that testing sessions were rather long and testing was repetitious.

3. Analyses

The above procedures thus resulted in measurements of 14 primary mental abilities on each of 10 occasions for each of 106 subjects. These data may be regarded as set out in the data box shown in Figure I. The grids within this box define cells in which are located separate scores; the depth, width and length dimensions of the box correspond respectively to the number of subjects, the number of primary variables and the number of occasions.

Notice that the grids within the data box mark off matrices. It is helpful to refer to these as files which might be drawn out separately, as from a file cabinet. Thus the front face of the box, representing the first front-vertical (as opposed to side-vertical) file, would, in this study, be a 106-by-14 matrix containing primary factor measurements obtained on the first occasion--a matrix of the kind involved in a typical R-technique single-occasion factor analysis. If the box is approached from the top, the first horizontal file is a 10-by-14 matrix such as would be used in a P-technique analysis for one subject.

Product-moment intercorrelations between primary factors were obtained separately for each of the 10 occasions--i.e., on each of the 10 front-vertical files described above. The resulting correlation matrices (symbolized R_k, where $k = 1, 2, \ldots, 10$) were added together and the sum was divided by 10 to produce the within-sessions, Rs, matrix shown in Table 2. Levin (1966) has recently pointed out that the factors of this matrix provide one kind of best-estimate of factors to reproduce the individual R_k matrices which go into the sum.

To obtain the correlation matrix shown in Table 3, each subject's standard scores on a particular primary were summed over the 10 occasions and the resulting total scores for the primaries were intercorrelated. The matrix obtained by these operations is the T_1 matrix referred to in Section III of this report. It will be recalled that according to the reasoning advanced in that section, T_1 provides the principal statistics upon which to base the definition of factors representing traits.

The W_1 correlation matrix shown in Table 4 was obtained from the horizontal files taken by approaching the data box from the top. That is, the 14 primary factors were intercorrelated for each subject separately, using the 10 occasions as the sample. The resulting 106 correlation matrices were summed and the result was divided by 106. It

Table 2

S_1

Intercorrelations Among 14 Ability Primaries. Within-Sessions Matrices Pooled Over 10 Occasions

PRIMARY FACTOR

		P	Ai	Vz	Cf	CFR	D	Ms	I	CMR	O	Fa	Fi	V	Fr
P	P	-	18	27	19	11	12	08	12	10	03	14	13	03	04
R	Ai	-	-	35	31	19	20	28	22	15	12	20	25	11	13
I	Vz	-	-	-	40	22	21	17	23	16	08	15	21	12	12
M	Cf	-	-	-	-	31	31	29	34	32	19	31	32	24	14
A	CFR	-	-	-	-	-	23	16	29	20	11	16	15	09	08
R	D	-	-	-	-	-	-	12	35	20	25	20	19	23	08
Y	Ms	-	-	-	-	-	-	-	25	21	09	23	16	14	09
	I	-	-	-	-	-	-	-	-	28	25	33	29	24	12
F	CMR	-	-	-	-	-	-	-	-	-	20	35	23	37	18
A	O	-	-	-	-	-	-	-	-	-	-	21	22	31	00
C	Fa	-	-	-	-	-	-	-	-	-	-	-	50	36	18
T	Fi	-	-	-	-	-	-	-	-	-	-	-	-	19	26
O	V	-	-	-	-	-	-	-	-	-	-	-	-	-	09
R	Fr	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 3

T_1

Intercorrelations Among 14 Ability Primaries
For Scores Obtained by Summing Over 10 Sessions

PRIMARY FACTOR

		P	Ai	Vz	Cf	CFR	D	MS	I	CMR	O	Fa	Fi	V	Fr
P	P	-	35	51	31	29	26	18	26	18	02	24	25	09	12
R	Ai	-	-	57	51	43	35	33	41	31	20	30	43	16	27
I	Vz	-	-	-	54	46	43	33	44	32	12	27	32	21	24
M	Cf	-	-	-	-	62	61	45	65	62	32	49	50	39	27
A	CFR	-	-	-	-	-	59	43	62	43	19	33	30	18	14
R	D	-	-	-	-	-	-	27	73	53	45	42	35	38	19
Y	Ms	-	-	-	-	-	-	-	48	44	11	38	25	26	22
	I	-	-	-	-	-	-	-	-	66	45	62	52	48	28
F	CMR	-	-	-	-	-	-	-	-	-	39	73	48	76	36
A	O	-	-	-	-	-	-	-	-	-	-	36	35	54	-03
C	Fa	-	-	-	-	-	-	-	-	-	-	-	77	61	57
T	Fi	-	-	-	-	-	-	-	-	-	-	-	-	37	65
O	V	-	-	-	-	-	-	-	-	-	-	-	-	-	65
R	Fr	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 4

W_1

Intercorrelations Among 14 Ability Primaries
Within-Subjects Matrices Pooled Over 106 Subjects

PRIMARY FACTOR

		P	Ai	Vz	Cf	CFR	D	Ms	I	CMR	O	Fa	Fi	V	Fr
P	P	-	03	04	06	00	01	01	02	04	04	04	02	-02	01
R	Ai	-	-	11	-01	00	04	03	01	03	04	09	03	01	02
I	Vz	-	-	-	23	00	00	01	05	04	00	03	07	01	03
M	Cf	-	-	-	-	02	-04	04	-02	06	04	04	05	03	03
A	CFR	-	-	-	-	-	-06	-04	07	06	06	01	02	01	00
R	D	-	-	-	-	-	-	-05	03	-06	06	01	03	07	00
Y	Ms	-	-	-	-	-	-	-	01	-01	05	04	04	-01	-06
	I	-	-	-	-	-	-	-	-	01	08	06	04	-01	-01
F	CMR	-	-	-	-	-	-	-	-	-	05	02	01	03	03
A	O	-	-	-	-	-	-	-	-	-	-	06	04	04	03
C	Fa	-	-	-	-	-	-	-	-	-	-	-	10	05	-22
T	Fi	-	-	-	-	-	-	-	-	-	-	-	-	-04	-09
O	V	-	-	-	-	-	-	-	-	-	-	-	-	-	04
R	Fr	-	-	-	-	-	-	-	-	-	-	-	-	-	-

will be recalled that according to the rationale advanced in Section III, W_1 provides the principal statistics upon which to base the definition of factors representing functional unities in change within persons--i.e., either states or patterns of function fluctuation of traits.

In forming B_1 the inverse of W_1 was scaled by pre- and post-multiplying it with a diagonal matrix containing the reciprocals of the square roots of the sums of squares of W_1^{-1}. This normalization was done because the diagonal elements of W_1^{-1} were slightly different from 1.0--a condition which, in general, would produce lack of symmetry in a product-matrix involving W_1^{-1}--and it was desirable to keep B_1 symmetrical. The result obtained by multiplying T_1 by the scaled W_1^{-1} was itself scaled to produce a matrix similar to a correlation matrix. This result is shown in Table 5. According to the rationale presented in Section III, this matrix has been "purged," as it were, of within-subject variability which otherwise could distort the identification of trait patterns.

Each of the matrices presented above was factored by an iterative principal axes procedure. Unities were retained in the principal diagonals of the matrices. Applying what Horn (1965) has referred to as the Kaiser-Guttman-Dickman (KDG) criterion for determining the number of reliable common-factors, no more than four factors was indicated for S_1, T_1 and B_1; accordingly, four factors were extracted in all analyses. The principal axes factors were rotated to achieve approximation to simple structure, first using Varimax (Kaiser, 1958) and then using the Promax (Hendrickson and White, 1964) procedure with power set at three. The results from these analyses are shown in Tables 6 and 7.

The difference between S_1 and B_1 is shown in Table 8. As previously argued, this represents the covariation on occasions which is not accounted for by the stable trait covariation represented in B_1. This within-sessions residual was factored using the same kind of procedures as were described above for other analyses: the results are shown in Table 9.

The principal results from the analyses of this study are presented in summary form in Table 10. In this table similar promax-rotated factors from the different analyses have been grouped together and only those variables having factor coefficients larger than .25 for a given factor have been listed to characterize that factor. At the foot of each table is listed the percent of total variance accounted for by the separate factors.

Table 5

B_1

Stable Trait Relationships Among 14 Ability Primaries, as Obtained by the Discriminant Procedure

PRIMARY FACTOR

		P	Ai	Vz	Cf	CFR	D	Ms	I	CMR	O	Fa	Fi	V	Fr
P	P	-	27	49	20	20	15	10	15	07	-06	13	15	00	04
R	Ai	-	-	49	44	34	22	23	29	16	12	13	29	04	14
I	Vz	-	-	-	22	25	21	16	19	07	-02	02	06	05	06
M	Cf	-	-	-	-	53	53	33	58	51	29	35	37	29	14
A	CFR	-	-	-	-	-	56	43	59	37	11	27	25	09	11
R	D	-	-	-	-	-	-	33	80	57	41	43	34	35	19
Y	Ms	-	-	-	-	-	-	-	52	46	07	39	25	25	24
	I	-	-	-	-	-	-	-	-	61	39	55	44	44	21
F	CMR	-	-	-	-	-	-	-	-	-	37	69	42	75	29
A	O	-	-	-	-	-	-	-	-	-	-	08	12	38	-21
C	Fa	-	-	-	-	-	-	-	-	-	-	-	77	51	66
T	Fi	-	-	-	-	-	-	-	-	-	-	-	-	32	63
O	V	-	-	-	-	-	-	-	-	-	-	-	-	-	08
R	Fr	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 6

Varimax-Rotated Factors Derived From S_1, T_1, W_1 and B_1

		Second-order Factors From																			
		S_1					T_1					W_1					B_1				
		I	II	III	IV	h^2	I	II	III	IV	h^2	I	II	III	IV	h^2	I	II	III	IV	h^2
P	P	-03	02	78	10	62	00	03	80	09	65	09	10	26	04	09	00	01	78	09	61
R	Ai	54	00	34	18	45	31	07	65	22	57	-05	37	26	15	23	34	00	66	08	55
I	Vz	40	05	62	10	55	32	08	77	11	71	-06	05	72	05	53	12	00	85	-02	73
M	Cf	52	30	32	22	51	60	35	41	22	71	02	-12	70	02	51	65	28	28	12	59
A	CFR	55	20	15	-07	38	74	15	37	00	72	69	-16	-06	-04	51	85	-05	18	04	77
R	D	23	56	30	-07	46	50	53	36	01	67	-17	71	-11	-01	54	71	41	10	13	70
Y	Ms	75	00	-18	18	63	78	00	02	23	66	-13	-05	14	32	14	62	05	02	28	46
	I	42	48	12	14	45	62	52	27	24	78	43	21	-04	14	25	76	42	10	24	82
F	CMR	26	44	-06	39	42	47	61	06	42	77	45	-13	23	-08	28	40	68	01	42	80
A	O	00	72	03	00	52	01	85	09	-02	74	44	42	05	06	38	21	75	-02	-25	67
C	Fa	14	40	05	63	58	26	48	10	74	85	12	14	02	67	48	26	32	01	85	90
T	Fi	10	25	21	67	57	11	31	30	78	80	10	00	15	45	23	21	18	14	79	73
O	V	04	65	-10	28	51	15	80	-04	28	74	09	43	04	-13	21	06	84	00	30	79
R	Fr	07	-15	05	70	53	11	-08	12	89	82	06	14	21	-71	57	10	-16	03	88	82
Variance		194	207	144	174	719	272	277	230	243	1022	117	115	132	133	497	308	231	191	264	994
% Total Variance		139	148	103	124	514	194	198	164	174	730	084	082	094	095	355	220	165	136	189	710

Table 7

Promax-Rotated Factors and Intercorrelations Among Factors Derived From S_1, T_1, W_1, and B_1

		Second-order Factors From															
		S_1				T_1				W_1				B_1			
		I	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
P	P	-20	-03	77	10	-15	-03	77	06	07	07	25	00	-12	03	76	07
R	Ai	44	-13	26	09	16	-06	55	13	-10	35	25	09	23	-06	58	01
I	Vz	24	-06	55	03	15	-05	66	02	-11	00	72	-05	00	00	81	-06
M	Cf	37	17	21	10	44	16	23	05	00	-17	71	-07	51	15	16	00
A	CFR	47	11	05	-18	64	-04	19	-17	71	-22	-11	-01	80	-22	01	-08
R	D	07	50	23	-16	35	38	20	-16	-22	73	-14	-03	57	25	-01	-01
Y	Ms	75	-13	-30	04	72	-19	-17	10	-15	-06	16	29	54	-09	-10	18
	I	30	39	02	01	45	32	08	05	41	17	-09	14	60	24	-03	08
S	CMR	17	36	-14	30	29	45	-12	26	45	-18	19	-09	19	56	-04	30
Y	O	-14	71	-02	-07	13	82	04	-14	41	36	00	03	10	70	-01	-35
M	Fa	01	31	00	56	07	33	-04	61	08	10	00	65	05	23	-04	77
B	Fi	-04	16	17	62	08	18	19	69	08	-03	15	42	02	11	09	73
O	V	-06	61	-15	20	00	72	-14	14	06	42	00	-16	-15	79	03	19
L	Fi	04	-23	05	69	00	-20	03	85	07	14	19	-74	-01	-22	-02	87
	I	-	40	40	30	-	47	46	41	-	19	16	05	-	42	32	36
	II	-	-	24	25	-	-	28	35	-	-	16	10	-	-	04	25
	III	-	-	-	11	-	-	-	25	-	-	-	15	-	-	-	13
	IV	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 8

The Difference Between S_1 and B_1

PRIMARY FACTOR

		P	Ai	Vz	Cf	CFR	D	Ms	I	CMR	O	Fa	Fi	V	Fr
P	P	-	09	22	01	09	02	01	03	-03	-09	-01	02	-02	00
R	Ai	-	-	14	14	15	02	-05	07	00	00	-07	05	-07	00
I	Vz	-	-	-	-18	04	00	-02	-04	-09	-09	-13	-15	-07	-05
M	Cf	-	-	-	-	23	22	05	24	19	10	04	05	05	00
A	CFR	-	-	-	-	-	33	27	30	17	00	10	09	00	03
R	D	-	-	-	-	-	-	21	45	37	16	23	15	13	11
Y	Ms	-	-	-	-	-	-	-	28	26	-02	17	09	11	15
	I	-	-	-	-	-	-	-	-	33	14	22	15	20	09
F	CMR	-	-	-	-	-	-	-	-	-	17	34	18	38	12
A	O	-	-	-	-	-	-	-	-	-	-	-13	-10	08	-21
C	Fa	-	-	-	-	-	-	-	-	-	-	-	27	15	48
T	Fi	-	-	-	-	-	-	-	-	-	-	-	-	12	37
O	V	-	-	-	-	-	-	-	-	-	-	-	-	-	01
R	Fr	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 9

Factors in Within-Session Residual, B_1 Trait Influences Having Been Removed

Second-Order Factors

		Varimax Solution					Promax Solution			
		I	II	III	IV	h^2	I	II	III	IV
P	P	13	04	63	03	42	09	06	62	02
R	Ai	59	-14	19	00	40	58	-20	13	-01
I	Vz	-02	-01	79	-15	64	-07	04	79	-16
M	Cf	62	17	35	-05	55	64	08	37	-07
A	CFR	60	32	17	07	50	57	25	16	03
R	D	36	62	00	07	51	33	56	04	02
Y	Ms	06	49	15	18	30	02	47	19	14
	I	35	63	-02	06	52	32	58	03	01
F	CMR	-01	75	-12	11	58	-04	73	-04	06
A	O	06	31	-32	-51	47	08	33	-26	-54
C	Fa	-07	36	-09	68	69	-09	30	-09	65
T	Fi	10	15	-14	62	44	10	07	-16	61
O	V	-34	61	-06	-04	50	-37	65	04	-07
R	Fr	00	05	00	81	66	00	-02	-05	81
Variance		151	236	138	186	711				
% Total Variance		108	169	098	133	507				

Correlations

I	-	16	-14	15
II	-	-	12	07
III	-	-	-	06
IV	-	-	-	-

Table 10

Summary of Promax-Rotated Results From Different Analyses

		Fluid Intelligence From						Crystallized Intelligence From				
		B_1	T_1	S_1	W_1	S_1-B_1		B_1	T_1	S_1	W_1	S_1-B_1
P	CFR	80	64	47	71	57	V	79	72	61	42	65
R	I	60	45	30	41	32	O	70	82	71	36	33
I	D	57	35	07	-22	33	CMR	56	45	36	-18	73
M	Ms	54	72	75	-15	02	D	25	38	50	73	56
A	Cf	51	44	37	00	64	I	24	32	39	17	58
R	CMR	19	29	17	45	-04	Fa	23	33	31	10	30
Y	Ai	23	16	44	-10	58						
		22	19	14	08	11		16	20	15	08	17

		General Visualization From						General Fluency From				
		B_1	T_1	S_1	W_1	S_1-B_1		B_1	T_1	S_1	W_1	S_1-B_1
P	Vz	81	66	55	72	79	Fr	87	85	69	-74	81
R	P	76	77	77	25	62	Fa	77	61	56	65	61
I	Ai	58	55	26	25	13	Fi	73	69	62	42	65
M	Cf	16	23	21	71	37	CMR	30	26	30	-09	06
A							Ms	18	10	04	29	14
R							O	-35	-14	-07	03	-54
Y		14	16	10	09	10		19	17	12	09	13

V. DISCUSSION OF RESULTS

This study was premised on an assumption that six factors, representing six basic functions in intellectual performance, could appear at the second order. Yet the analyses described above led to the identification of only four factors. How is the discrepancy between this possible outcome and the actual outcome to be interpreted?

First, it must be noted that one of the major weaknesses of dimension-identifying procedures of the kind employed in this study is that they do not include adequate tests for determining the number of reliable, replicatable dimensions: such tests have yet to be developed. Hence the decision concerning the number of factors to extract in this study is, in a sense, arbitrary. Therefore, the fact that only four factors were extracted should not be taken to mean that an hypothesis stipulating more (or fewer) factors must (necessarily) be rejected.

But while the decision to extract four factors is arbitrary in the above-mentioned sense, it is not entirely without foundation. In fact, the decision is based upon a widely-used rationale (see Horn, 1965)--a rationale that is regarded with favor by many who use and write about dimension-identifying procedures. It provides an objective-analytic (as opposed to subjective-judgmental) basis for determining the number of factors and such was deemed desirable at this exploratory stage in the study of new methods. Later, as the methods employed in this study become better understood, it may be worthwhile to decide the number-of-factors question in a more subjective manner.

Finally, too, it must be noted that although, theoretically, and on the basis of previous findings, six factors could be expected at the second-order, the sampling of primary factors for this study was not such as to make it likely that six factors would, in fact, be indicated by analytic tests. Of the markers previously employed to identify Gs, only P was included in the present study and not one of the markers previously employed to identify C was included. It was thought that Fr would represent the major variance on C, that P would provide substantial variance on Gs and that other primaries would contribute enough variance to allow for identification of these two second-order factors. But in retrospect it can be seen that these assumptions were not justified, assuming that analytic criteria would be used to determine the number of factors. More primaries to produce variance on Gs and C should have been included to ensure that the roots corresponding to these two factors would be greater than unity. Hence, on these grounds it can be argued that the four factors obtained in this study probably do represent the reliable common variance of the primaries sampled. The fact that the other second order factors were not reliably defined indicates that the sampling of primary factors was not broad enough to permit this.

But while Gs and C were not adequately represented in this study, it is noteworthy that factors representing each of the other hypotheses appear in all analyses, both those directed at revealing state-like patterns and those directed at revealing trait-like patterns. This suggests that all four of the major functions--fluid intelligence (Gf), crystallized intelligence (Gc), general visualization (Gv) and general fluency (F)--have state-like and trait-like properties. However, it is noteworthy, too, that the patterns defined by different analyses differ in several respects and that some of these differences are quite pronounced.

It might seem at first that the differences in patterns revealed by different analyses represent only variation in rotation procedures. However, it should be noted that none of the rotations involve subjective procedures (cf. Horn, 1967) and that all are based upon exactly the same analytic criteria. Also, judging by results from studies comparing rotational procedures (see Horn, 1963a), some of the differences between factor patterns are simply too large to be attributable to variation in rotational techniques. Other explanations for the differences must be sought.

Before considering differences in the patterns revealed by these analyses, it is perhaps worthwhile to first briefly consider some of the similarities in these results and to focus upon some of the general characteristics of the solutions.

In this respect notice that in all analyses three of the second-order factors (excluding Gv) are defined by prominent loadings on at least three primaries common to the factor in more than one analysis and that for Gv this is true for at least two primaries. It is on this basis that we can effectively argue that the same, somewhat independent processes are indicated in all analyses. Since the analyses on W_1 are based exclusively on within-person variation and covariation, while the analyses on T_1 and S_1 represent only between-person variation and covariation, these results show that the general abilities defined in this study have the status of functional unities, as this concept is defined by Cattell (1957) and in general biological science. That is to say that just as the somewhat independent functions of the heart and the liver are represented by distinct patterns of variation of several physiological measurements, as revealed both by analyses of this variation within persons and by analysis of variation between persons, so somewhat independent intellectual functions are indicated in this study by analyses of both within-person and between-person variation in performance on intellectual tasks. Cattell (1950; 1957) has cogently argued that this kind of evidence on the functional nature of factors is essential if we are to gain a truly adequate description of personality. But this is the first study in which this kind of evidence has been presented for human ability attributes and it is one of a very few studies presenting evidence of this kind for any behavioral attribute.

In all analyses, except that on S_1-B_1, fluid intelligence is defined by CFR, figural relations, I, induction, and CMR, semantic relations. The essential processes implied by these tasks would still seem to be well-described in Spearman's (1927) penetrating discussion as the perception of relations and the eduction of correlates. According to the refinements introduced in the theory of fluid and crystallized intelligence, these essential processes are best revealed in Gf when test materials are such that they indicate mainly reasoning, abstracting, span of awareness, etc. (cf Horn & Cattell, 1966a) in the immediate testing situation, rather than as distilled from past experience. In this respect it is noteworthy that the three primaries mentioned above as defining Gf in this study are three of the same primaries which defined the factor identified as Gf in the studies preceding this one. Thus, the more detailed discussions of process in previous studies can be taken as applicable to the results from this study (see Horn, 1965a for the most complete treatment of this topic).

In all analyses of this study, Gc is identified by V, verbal comprehension, O, originality, and D, deduction. This is particularly interesting in that D is not patently a verbal primary and O has heretofore been treated as mainly an indication of creativity, conceived of as independent of intelligence.[5] Thus the essential process, particularly when considered in terms of state variability within a person, would appear to be eduction of correlates, as in Gf; but here, as in previous studies in which Gf and Gc have been distinguished, the eduction of Gc (relative to that of Gf) is seen to depend much more upon the "esotericness" of the experience which has been, as it were, put into the person. Thus in O, particularly, the wider a person's experience, the more likely that he can derive a correlate to represent a relationship among three concepts sampled from a very wide range of concepts (see Mednick, 1963; Mednick & Mednick, 1963; Mednick, Mednick & Jung, 1966). If a fuller description of process in this factor is desired, it is worthwhile to refer to Horn (1965a) or Horn & Cattell (1966a).

Although the loadings on some primaries are not high, the general visualization function is clearly revealed in all analyses except, perhaps, that in which trait influences were subtracted out of the within-sessions matrix. In all analyses V_z, visualization, and P, perceptual speed, have prominent loadings; Ai, aiming, is prominent in analyses emphasizing trait variability. Interestingly, Cf, flexibility of closure, is not very prominent in analyses for traits, but is prominent in analyses for states. Overall, the major processes would still seem to be those of imagining change in space, finding a Gestalt, maintaining flexibility concerning various possible structurings of elements in space, etc., as described fully by Horn (1965a) and Horn & Cattell (1966a).

General fluency is defined in all analyses by Fa, fluency of association, and Fi, ideational fluency. In analyses for trait it has variance on CMR, semantic relations, whereas in analyses for state there appears to be a low relationship to Ms, memory span. In terms of the hypotheses presented in Section II, these results suggest that the <u>trait</u> of general fluency depends to some extent upon the size of the store of concepts in memory, as represented by CMR. As suggested by interpretations of this factor in previous studies, however, trait F apparently does not depend upon the clarity of perception of differences among concepts; indeed, O, representing this function, has a negative relationship to F in all but the W_1 analysis, and this relationship is substantial in the B_1 and $(S_1$-$B_1)$ analyses. The state variation in F, on the other hand, does not involve CMR or O, but does involve immediate memory. This suggests that function fluctuation on this factor may involve speed of transmission from long-term memory to expression modalities.

Turning now to consideration of general characteristics of the solutions, we notice that the factors defined in the T_1 and B_1 matrices are, in general, broader than the factors defined by other analyses--broader in the sense that they involve more variables with larger loadings. The factors defined in W_1 are least broad in this sense. The percent of total variance accounted for by the common variance of four factors is largest for T_1, drops off for the other solutions in the order B_1, S_1, S_1-B_1, and is smallest for W_1. This suggests that the most general kinds of constructs, involving both trait and state variability, are defined in observations combined over several occasions in which a person (or other organism) might be observed. On the other hand, the fact that the common variance defined by four factors is smallest for W_1 indicates that much of the variation within a person in performances on ability tasks is unsystematic--i.e., error.

This latter is not an unexpected finding, of course. Indeed, it would seem that in much previous research on (and discussion of) abilities it has been assumed, albeit implicitly, that all variation within the person is random. That this assumption is not warranted is indicated in the present results by the fact that the four functions (Gf, Gc, Gv and F) are defined by distinct patterns of covariance in W_1. These patterns are too consistently in line with other findings to make it reasonable to suppose that they represent only random variance.

That the common variance of four factors in B_1 is less than the similar common variance in T_1 is consistent with the hypothesis that T_1 represents both trait and state variance, while B_1 represents trait variance alone. In most other respects, however, the solution obtained on T_1 is equivalent to that obtained on B_1. That is to say that even when considered in terms of details, the interpretations of factors derived from T_1 would, as far as the evidence at hand is concerned, need to be nearly the same as interpretations based upon analysis of B_1. There are a few small differences. For example, there is a suggestion in these data that none of the processes represented by Vz is truly characteristic of the trait of fluid intelligence, although such a process appears to be involved when analyses exclude between-occasion variance (presumably this outcome represents the fact of substantial correlation between Gv and Gf). However, a sceptic might be inclined to question the replicability of detailed findings of this kind. He could argue that the small differences between B_1 and T_1 factors represent nothing more than slight variations in testing conditions, unusual behaviors of certain subjects, rounding errors and somewhat different iteration cycles in computation or other such trivial (though perhaps systematic) influences. Taking this conservative position, it would seem that with data of the kind analyzed in this study, it is reasonable to suppose that results from analyses of total scores (obtained by summing over occasions) will be highly similar to results obtained by a discriminant analysis to reveal "pure" traits.

The discrepancies between the results from analyses on S_1 and analyses on B_1 are somewhat larger than those for T_1 and B_1. In particular, for example, D would not be said to help define Gf if only the results on S_1 were obtained, whereas it would be said to be in the Gf pattern if the results from analyses on B_1 or T_1 were used. A similar condition holds with respect to the presence of O in the general fluency dimension, and somewhat smaller discrepancies of this kind exist for other of the relationships. But although these discrepancies are larger than those indicated in comparison of B_1 with T_1, they are, for the most part, small relative to the kinds of discrepancies seen in attempts to replicate factorial findings in separate studies using the same methods but different samples of subjects. Hence on this basis the sceptic might well argue that essentially the same results are indicated by the analysis on S_1 and those on B_1 or T_1.

Although this argument certainly appears to be applicable to the results obtained in this study, it is not generally applicable. In fact, the similarity in results obtained on S_1 and T_1 indicates a finding that the Gf, Gc, Gv and F functions are mainly trait-like rather than state-like. As pointed out in Section III, if attributes are mainly state-like, the T_1 matrix will approximate an identity matrix and the S_1 matrix will have non-zero off-diagonal elements indicating the extent of common variation of the several variables defining the states. Insofar as these conditions do not hold, and the patterns of covariation on each given occasion are similar to the patterns of covariation among scores totaled over all occasions, the patterns of covariability observed on each given occasion can be taken to be indicative of traits. The evidence of the present study is thus clearly in support of an hypothesis stipulating that the broad ability functions of this study represent stable traits--at least stable over the time span and range of measurement conditions here considered.

These conclusions are quite consistent with those deriving from consideration of the small common variance indicated in W_1. But the analyses on W_1 put these conclusions in a somewhat different light and reveal features that are not shown by the other analyses.

Of the five primaries which have substantial correlations with the trait component of fluid intelligence (as represented in B_1), only three (CFR, I and CMR) show reliable within-person variation related to this function (as disclosed in W_1). In addition O, which does not fall into the trait pattern, is clearly present in the state pattern. Contrary to expectations, the state variability of Gf is not highly associated with span of apprehension, as represented in Ms. Similarly this state variability is not associated with fluctuations of visual perceptiveness, as represented in Cf, Ai and D. Thus the evidence of the W_1 analyses suggests that the dynamic quality of Gf is to be understood in terms of the reasoning and relation-perceiving processes demanded in identifying relations among figures (CFR), identifying relations among verbally-tagged concepts (CMR), producing correlates to continue a series (I) and producing correlates to illustrate a relation among verbally-tagged concepts (O).

In the crystallized intelligence function the dynamic quality is involved most prominently in D, indicating deductive processes, but is shown also in recalling and perceiving relations among culturally-established concepts, as represented in V and O.

The fact that O appears in the dynamic components of both Gf and Gc and the fact that the former is characterized by I, induction, while the latter is characterized by D, deduction, suggests that an important aspect of the function fluctuations of Gf and Gc may have to do with choice of a reasoning strategy for attacking a problem. In the Bruner, Goodnow and Austin (1956) work on strategies in thinking it is pointed out that an individual may for a while favor one approach to a class of problems and then change to another approach. Relatedly, Horn & Cattell (1966a) define a concept of alternative mechanisms to represent the fact that some problems may be solved by proper use of either one of two distinct abilities. Thus the problems of O, when considered in terms of dynamic variability within the person, can be solved by exercise of fluid intelligence processes or by exercise of crystallized intelligence processes. In the former there is emphasis on induction using only the evidence of the immediate situation whereas in the latter there is emphasis on deduction using evidence recalled from previous experience. But in exercising the former processes, one might tend to exclude use of recall-deductive processes, and vice versa. Thus the fact that the major state variability in Gf and Gc is shown in immediate-inductive and recall-deductive processes respectively suggests that the individual may be shifting back and forth in his dependence upon one or the other of these processes.

In Gv the dynamic (state) pattern is very closely parallel to the static (trait) pattern, except that the order of variables according to size of loadings is almost perfectly opposite for the two. Contrary to what might have been expected, perceptual speed (P) and aiming (Ai) have the smallest state variance and nearly the largest trait variance, whereas flexibility of closure (Cf) has nearly the largest state variance and the smallest trait variance. The visualization (Vz) primary has about the same variance in the state factor as in the trait dimension. These results, overall, suggest that state variability in the general visualization function is shown most clearly when tasks require one to imagine changes in space and to employ visual tracking. These processes are to be contrasted with what might be characterized as focusing or fixing on a particular pattern. In both P and Ai the task requires this fixing on a pattern, whereas in Cf and Vz the task requires that one allow the eye to follow in and around the curves of a pattern.

A rather interesting reversal of relationship is indicated for the general fluency function. The number of associations judged to be irrelevant is negatively related to the dynamic F pattern based upon analysis of W_1, but is positively related to the trait pattern. What this may mean is that: (1) the person who characteristically gives many associations tends, relative to other people, to give rather many irrelevant associations, and (2) now considered in terms of variability within a person, whenever a person gives a relevant association, he is thereby not giving an irrelevant association, so that as the number of one kind of association increases, the number of the other kind must decrease.

This finding is interesting not only because it provides some information about the general fluency function, but also because it illustrates something about the research strategy employed in this study. The reversal finding could not have been discovered by R-technique analyses alone; if the negative association were found in a P-technique analysis which followed several R-technique studies (as would normally be the case), then it could very well be misinterpreted as an anomalous lack of replication of previous results. But in the present study the opposite associations are clearly shown to exist in the same data, so the negative association cannot be interpreted as merely an anomaly resulting from drawing a rather odd (one person) sample. When seen in this way the opposite-sign relationships provide useful information about the nature of the concepts under investigation.

According to the rationale outlined in Section III of this report and stated more fully in the Horn-Little (1966) article, the S_1-B_1 residual represents state variance and variance associated with situations but not fully stable over all situations. This latter is a rather awkward concept, but it indicates a kind of variability which is distinct from that represented by the concepts of state and trait. Consider an example which illustrates this.

In the present study, five of the ten testing sessions occurred in the morning and five in the afternoon. Now it is possible that some individuals are quite different from others in the ways in which they react to attempts to measure their abilities in the morning as compared with the afternoon. Early-risers, for example, may be "set to go" at an early hour while late-risers have yet to become fully awake. Suppose that two kinds of performance representing a functional unity vary concomitantly for both early-risers and late-risers, but a third performance varies concomitantly in this pattern only for people who are fully awake and a fourth performance covaries with the others in the pattern only for people who are half asleep. Thus in afternoon sessions, when both early-risers and late-risers are fully awake, variable three will contribute considerable variance to the pattern in question, but in morning sessions it will contribute less variance, whereas variable four will contribute some variance to the pattern in morning sessions but none in the afternoon. This would mean that some of the covariance observed in separate sessions would be associated with sessions, per se; it would not represent stable trait, characterizing the person despite variation over occasions, and not all of it would be picked up as state variance since (by definition) it would vary with state on only one-half of the occasions.

The Gf factor defined in the analysis by S_1-B_1 differs from the trait pattern in three noteworthy respects: Ms and CMR are missing from the factor and Ai has an unexpectedly large loading in it.

The fact that Ms is missing is understandable if it is assumed that almost all of the reliable Gf variance on this primary is trait, for this variance was presumably subtracted out. This is consistent with the finding that Ms has virtually no variance in the state patterns for Gf.

But while this kind of explanation is perhaps reasonable for the finding with respect to Ms, it does not apply to the results obtained for CMR. This latter has substantial loading in the state pattern of Gf. Hence if only trait variance were removed in S_1-B_1, the Gf pattern determined on this should contain some state variance on CMR. The fact that it doesn't suggests that analyses on S_1-B_1 are not ideally suited to reveal patterns of covariation corresponding to states.

This last conclusion is supported by the fact that, in general, the patterns revealed in analyses on S_1-B_1 are more similar to the findings from analysis for traits than they are to the factors defined on W_1. It is noteworthy in this respect that Fr does not have a negative correlation with the general fluency dimension defined in S_1-B_1, while it does, as noted above, correlate negatively with the state pattern of general fluency. It seems, therefore, that the covariance of S_1-B_1 is mainly indicative of individual differences in reactions to subclasses of the situations in which measurements were obtained. Since it was previously supposed that analyses of S_1-B_1 would be mainly indicative of states, these conclusions indicate a need to modify the rationale upon which the analysis of S_1-B_1 was based. Such modification will not be attempted here, however.

Turning now to the question about the relative contributions of state and trait to the observed variability of the four intellectual functions, we may notice first that in the W_1 analysis to reveal states, general fluency and general visualization have the larger common variances and Gf and Gc the smaller ones. This is noteworthy because the reverse of this ordering for common variance contribution exists for the factors determined on the total scores of T_1. In other words, these findings suggest that although Gv and F are mainly trait-like, they involve relatively more function fluctuation (relative to the total variability in these kinds of performances) than do Gf and Gc.

The common state variance for Gf is slightly larger than that for Gc and its variance in the analysis of T_1 is somewhat smaller than that for Gc. This thus suggests that Gf involves a bit more function fluctuation than does Gc. However the differences here are not large, so one would not want to read too much into this conclusion. It is interesting in this respect that when T_1 is "purged" of state variance in deriving B_1, the Gf factor then resulting has considerably more common variance than the Gc factor. It appears that state variability in Gf to some extent obscures the fact that the Gf trait can be expressed in several primaries which are not mainly characteristic of this function. In other words, fluid intelligence, conceived of only as trait, may have wider influence in intellectual performances than it seems to have if analyses are based upon matrices representing both state and trait variability.

VI. SUMMARY

Measurements of 14 primary mental abilities were obtained on 10 separate occasions for 106 male adult inmates of a state penitentiary.

All correlations among the 14 primaries were determined within each session considered separately and the resulting matrices were pooled over the 10 sessions to provide a matrix labeled S_1. Within each subject considered separately a 14 by 14 matrix of correlations among the primaries and over occasions was formed and the resulting 106 matrices were pooled to provide a matrix designated W_1. The primary factor measurements for each session considered separately were converted to standard score form and the resulting score matrices were summed to provide a total score, over all sessions, for each subject on each primary. A 14 by 14 matrix of intercorrelations for these total primary factor scores was obtained. This was symbolized T_1. Applying principles derived from discriminant function analysis, a rationale was developed for maximizing trait variability relative to state covariability among a set of variables. This involved multiplying T_1 by the inverse of W_1. The result was designated B_1. It was reasoned that if the covariability of B_1 were subtracted from S_1, the resulting residual would represent state covariability and non-trait variance associated with subsets of sessions.

The S_1, W_1, T_1, B_1 and S_1-B_1 matrices resulting from the above-mentioned analyses were factored using an iterative principal axes procedure. These initial factors were rotated orthogonally according to the Varimax criterion and obliquely using the promax proceding with power set at three. Four common factors were estimated in all analyses. The resulting factorial solutions were examined in some detail to see what evidence they provided about the dynamic, state-like and static, trait-like characteristics of broad attributes of intellectual functioning.

The four rotated factors in each solution were found to be quite representative of four major dimensions of intellect found in previous research. The four are referred to in general theoretical treatments as fluid intelligence (Gf), crystallized intelligence (Gc), general visualization (Gv) and general fluency (F). The fact that the four could be identified in all analyses indicated that all had both the properties of dynamic states and the properties of stable traits. The major variance contribution on all factors was from trait influences, however. Reliable, common state variability was relatively more pronounced in Gv and F than in Gf and Gc, but such variability was somewhat more pronounced in Gf than in Gc.

The patterns of loadings in the factors defining states suggested that:

(1) The within-person variation in fluid intelligence function is associated primarily with identifying relations and extrapolating from a set of relations to produce correlates representing new instances of the relations.

(2) Contrary to expectations, the within-person variability in fluid intelligence is not closely associated with short-term memory processes.

(3) The within-person variability in crystallized intelligence is associated primarily with deductive reasoning processes, the recall of concepts previously acquired and the perception of relations among these.

(4) Dynamic variability in general visualization is more closely associated with primaries requiring roving eye movements and imagination of things not seen than with primaries requiring focusing on particular, pre-defined patterns.

(5) Within the person, change in ideational fluency is closely associated with change in associational fluency but change in the direction of producing somewhat irrelevant associations has a substantial negative relationship with this pattern. This is particularly interesting in view of the fact that in between-persons analyses, such as traditional R-technique factoring, irrelevant associations has substantial positive correlation with the general fluency dimension.

The above mentioned findings with respect to Gf and Gc led to an hypothesis that within-person fluctuation in these functions represents, in part, a shifting in strategies used to attack intellectual problems.

Contrary to expectations, analyses on the S_1-B_1 residual did not seem to provide much in the way of useful information about states. It appears that such analyses do provide information about patterns of covariation associated with subsets of measurement situations, however.

Perhaps the principal value of this study is to be found in the research strategy adopted. This was directed at defining functional unities in terms of several conceptions about what might constitute a "unity" and what is meant by "functional". The results illustrated how fluid intelligence (as well as other attributes of intellectual test behavior) varies functionally within persons and also represents a stable pattern of performances that distinguishes one person from another. This kind of finding could have considerable value in several fields of psychology.

VII. FOOTNOTES

1. Here is introduced a distinction between precision and reliability that is not always recognized in psychometric literature. Precision in this context refers to the number of discriminations made among the entities assessed with a particular measuring device. In measuring the stature of humans, for example, a ruler marked off in millimeters provides for greater precision of measurement than a ruler marked off in centimeters. In this example, it is apparent that reliability (in the sense of agreement in the measures obtained for the same entity on two different occasions) for the millimeter measurements need not be any higher than for the centimeter measurements; indeed, it might well be lower. Moreover, it is logically evident that if precision continues to be increased, there must come a time when reliability decreases. It's in this sense that precision can be said to be purchased at some cost to reliability, and vice versa.

2. It is possible, of course, that this kind of outcome would be identified with an analysis of variance. If an emotionality measure were used to stratify subjects in a stratified block design, for example, the indicated result could show up as a significant subjects-by-session interaction. However, the point is that the effect could be--which is not to say it necessarily would be--missed entirely in a between-groups kind of analysis.

3. To indicate the historical link between modern concepts of intelligence and Spearman's pioneering developments in this area, in which he referred to the general intellective factor as G (of g), fluid intelligence and crystallized intelligence are symbolized Gf and Gc respectively.

4. The gathering and analyses of these data were possible only because I received help and cooperation from many people. Mr. Harry Tinsley, Director of Institutions for Colorado, and Warden Donald Patterson provided general approval for the gathering of data at the prison. Mr. George Levy was instrumental in arranging for this approval; he provided general supervision in the administration and scoring of tests and in numerous other ways contributed to completion of the project. Assistant Warden Wyse, Captain Yeo and several guards did the rather complex work required to enable subjects (located in

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5. See Getzels & Jackson (1962); Guilford (1962); Mednick (1963); Mednick & Mednick (1963) and Taylor (1964) for highly regarded presentations of this viewpoint, but see Burt (1962) and Horn & Cattell (1966a) for criticisms of these views on creativity and intelligence.

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